Boundary conditions in quantum string cosmology

Mariusz P. Dąbrowski [*]

Institute of Physics, University of Szczecin, Wielkopolska 15, 70-451 Szczecin, Poland

Claus Kiefer[†]

Fakultät für Physik, Universität Freiburg, Hermann-Herder-Str. 3, D-79104 Freiburg,

Germany

(February 1, 2008)

Abstract

We discuss in detail how to consistently impose boundary conditions in quantum string cosmology. Since a classical time parameter is absent in quantum gravity, such conditions must be imposed with respect to intrinsic variables. Constructing wave packets for minisuperspace models from different tree-level string effective actions, we explain in particular the meaning of a transition between "pre-big-bang" and "post-big-bang" branches. This leads to a scenario different from previous considerations.

submitted to Physics Letters B

---

[*]E-mail: mpdabfz@uoo.univ.szczecin.pl

[†]E-mail: kiefer@phyq1.physik.uni-freiburg.de

String theory seems to be one of the best current candidates for a theory which unifies gravity with other interactions. Since it applies to energies of the order of the Planck scale, it attracts the interest of cosmologists who are interested in initial conditions for the universe very close to a classical singularity. Much interest has been focused recently on low-energy effective actions from string theory [1]. Such actions contain additional fields in the gravitational sector, in particular dilaton and axion fields.

One of the advantages of such an effective theory is the possibility of having a superinflationary phase $a(t) \sim (-t)^p$ ($t < 0, p < 0$), which is driven by the kinetic energy of the dilaton, and which is free from the fine-tuning problem usually present in potential energy-driven de Sitter or power-law inflation. One of the central features of string theory is its symmetry with respect to duality transformations [2]. For simple isotropic cosmologies this leads to the scale factor duality ($a \to 1/a$) [3] which, when combined with time reversal symmetry, results in new, duality-related solutions. Usually, one considers one of these solutions as describing a superinflationary accelerated expansion and the other one as describing a decelerated (presumably radiation dominated) expansion. However, the superinflationary phase emerges only for negative times ($t < 0$) and its decelerated duality-related branch is separated by a singularity in curvature and string coupling. A desirable scenario would be to have a superinflationary phase for negative times (the "pre-big-bang" phase) followed by a standard radiation dominated expansion (the "post-big-bang" phase). However, in view of the appearance of the singularity between the two phases, this does not seem to be easily achievable. One thus looks for possible mechanisms to overcome this "graceful exit problem" in string cosmology [4].

It has been proven as a new type of "no-go" theorem [5] that there is no way to connect classically the duality-related solutions and to overcome the "graceful exit problem" in the simplest models of string cosmology. With respect to this result, it seems that the classical scenario breaks down and that one needs to take quantum effects into account to avoid the singularity. This can be achieved, for instance, by adopting higher-order α' (inverse string tension) corrections to the tree-level effective action [1,6]. Such an approach, though

preliminary, has been presented recently in [7]. Another possibility is to apply a one-loop superstring effective action, for which there exists a large class of nonsingular solutions for a very broad range of the parameters given in [8].

Staying on the tree-level sector of the string effective action, the formalism of canonical quantum gravity has been applied to describe a quantum transition form the "pre-big-bang" phase to the "post-big-bang" phase through the singularity [9,10]. More precisely, in the minisuperspace comprising scale factor (a) and dilaton (ϕ), a solution to the Wheeler-DeWitt equation was found after imposing boundary conditions in the strong coupling regime $\phi \to \infty$. This solution was interpreted as describing a reflection in minisuperspace through the singularity.

Such an interpretation is, however, tight to the presence of an external time parameter. Being redundant already in the classical theory due to time-reparametrisation invariance, an external time parameter is completely absent from quantum gravity (see, for example, the careful discussion in [11,12]). A classical time parameter can only emerge as an approximate notion through some Born-Oppenheimer type of expansion scheme [13].

How, then, can the above "transition" be consistently dealt with in quantum cosmology? The choice of boundary conditions as well as the interpretation of the quantum cosmological wave function should refer only to *intrinsic* variables, i.e. variables which directly occur in the Wheeler-DeWitt equation. In this respect the hyperbolic nature of this equation for such models is particularly important [11]. The purpose of our paper is the presentation of a consistent quantum cosmological scenario along these lines. Instead of referring to an external time, we shall construct wave packets that represent classical trajectories in quantum cosmology. This has been successfully applied before in quantum general relativity [14]. Furthermore, we shall suggest to impose boundary conditions in the region of small scale factor.

In the following we shall first stick to the simple model where only a positive cosmological constant is present [9,10]. The main conceptual issues can be discussed clearly in this context. We shall then proceed to discuss an example which exhibits turning points in configuration

3

space.

Before starting with the details of our analysis, we would like to emphasise that it is not, in general, justified to quantise an effective action (which itself arises from a fundamental quantum theory). For example, one would certainly not invoke a quantisation of the "Euler-Heisenberg" effective action of QED. However, in so far as new fundamental fields arise from the fundamental theory (such as dilatons and axions), a quantisation of the effective action could capture some relevant features. It is with this reservation that we present the following investigation.

We start with the Wheeler-De Witt (WDW) equation from a tree-level low-energy string effective action for zero spatial curvature, which contains a dilaton potential similarly to [9,10]. (Quantum cosmology for string models was first studied in [15]. A comparison between Dirac quantisation and reduced quantisation was made in [16].) It reads

$$\hat{\mathcal{H}}\Psi \equiv \left[-\partial_{\bar{\phi}}^2 + \partial_{\beta}^2 - \lambda_s^2 V(\beta, \bar{\phi}) e^{-2\bar{\phi}}\right] \Psi(\beta, \bar{\phi}) = 0\,, \tag{1}$$

where

$$\beta \equiv \sqrt{3}\ln a\,, \tag{2}$$

$$\bar{\phi} \equiv \phi - 3\ln a - \ln\int\frac{d^3x}{\lambda_s^3} \tag{3}$$

are redefined variables, $V(\beta, \bar{\phi})$ is the dilaton potential, and $\lambda_s \equiv \sqrt{\alpha'}$ is the fundamental string-length parameter (it is assumed that the volume of three-space is finite).

We first consider the simplest potential which is just given by a positive cosmological constant, $V(\beta, \bar{\phi}) = \Lambda$, and look for a separable solution of the form

$$\Psi_k(\beta, \bar{\phi}) = e^{-ik\beta}\psi_k(\bar{\phi}) \tag{4}$$

for all $k \in \mathbf{R}$ [9,10]. The function $\psi_k(\bar{\phi})$ then obeys the effective equation

$$\left(-\partial_{\bar{\phi}}^2 + V_{eff}(\bar{\phi})\right)\psi_k(\bar{\phi}) = 0\,, \tag{5}$$

where the effective potential is given by

$$V_{eff} = -k^2 - \lambda_s^2 \Lambda e^{-2\bar{\phi}}. \tag{6}$$

Since V_{eff} is always negative, there are no classically forbidden regions in the effective sector (as specified by k) of this simple model and therefore no "turning points". (In the full theory, there are no classically forbidden regions, since the kinetic term is indefinite). Later we shall discuss other possibilities, where classically forbidden regions and turning points exist.

The general solution of (5) is given in terms of Bessel functions [10],

$$\psi_k(\bar{\phi}) = c_1 J_{+ik}(z) + c_2 J_{-ik}(z), \tag{7}$$

with $z \equiv \lambda_s \Lambda e^{-\bar{\phi}}$. In the strong coupling limit $\bar{\phi} \to \infty$ $(z \to 0)$ one has

$$\lim_{z \to 0} J_{\pm ik}(z) e^{-ik\beta} \sim e^{-ik(\beta \pm \bar{\phi})}. \tag{8}$$

In order to get a deeper insight into the problem and to discuss the correct boundary conditions we include a brief discussion of the classical solutions for the string effective action equations [3,6,9,10]. Because of the string duality-symmetry, one obtains various "pre-big-bang" and "post-big-bang" branches, but we shall discuss those which attracted most interest: the expanding accelerated (or superinflationary) "pre-big-bang" branch and the expanding decelerated "post-big-bang" branch, which are classically divided by a singularity [10]. These are given by

(+) $t < 0$ ("pre-big-bang")

$$\beta = \beta_0 - \ln \tanh\left(-\frac{\sqrt{\Lambda}t}{2}\right), \tag{9}$$

$$\bar{\phi} = \bar{\phi}_0 - \ln \sinh(-\sqrt{\Lambda}t), \tag{10}$$

(−) $t > 0$ ("post-big-bang")

$$\beta = \beta_0 + \ln \tanh\left(\frac{\sqrt{\Lambda}t}{2}\right), \tag{11}$$

$$\bar{\phi} = \bar{\phi}_0 - \ln \sinh(\sqrt{\Lambda}t). \tag{12}$$

These branches are related by the duality symmetry including time-reflection $\beta(t) \to -\beta(-t)$, $\bar{\phi}(t) = \bar{\phi}(-t)$.

Since the canonical momentum with respect to β is given by $\Pi_\beta = -\lambda_s e^{-\bar{\phi}}\dot{\beta} \equiv -k =$ constant, one can express "expansion" by $k = \lambda_s\sqrt{\Lambda}e^{-\bar{\phi}_0} > 0$. The canonical momentum with respect to $\bar{\phi}$ is given by $\Pi_{\bar{\phi}} = \lambda_s e^{-\bar{\phi}}\dot{\bar{\phi}}$. In the strong coupling regime $\bar{\phi} \to \infty$ it reads for the cases $(+)$ and $(-)$, respectively,

$$\Pi_{\bar{\phi}}^{(\pm)} \overset{\bar{\phi}\to\infty}{\to} \pm\lambda_s\sqrt{\Lambda}e^{-\bar{\phi}_0} = \pm k. \tag{13}$$

A distinction between "expanding" and "contracting" has no intrinsic meaning, however, since we can arbitrarily change the sign of $\dot{\beta}$ after re-introducing the lapse-function. In quantum cosmology, where t is fully absent, this becomes even more evident, since no reference phase $\exp(-i\omega t)$ is available, with respect to which solutions could be classified as, e.g., right-moving or left-moving. This is in full analogy to the situation in ordinary quantum cosmology [11,17,18].

To make the identity of an expanding solution with a contracting solution explicit, it is more appropriate to discuss the string scenario in the configuration space formed by $(\beta, \bar{\phi})$. Eliminating t in (9)–(12), one finds that the trajectories in configuration space are given by

$$\beta = \beta_0 \pm \mathrm{arsinh}(Ke^{\bar{\phi}}) = \beta_0 \pm \ln\left[\left(K + \sqrt{K^2 + e^{-2\bar{\phi}}}\right)e^{\bar{\phi}}\right], \tag{14}$$

where the plus sign refers to the "pre-big-bang" branch $(+)$ and the minus sign to the "post-big-bang" branch $(-)$, respectively, and a new constant K,

$$K \equiv \frac{k}{\lambda_s\sqrt{\Lambda}} = \pm e^{-\bar{\phi}_0}, \tag{15}$$

has been introduced. (A change of sign of K corresponds to the change of the branch, the above distinction between $(+)$ and $(-)$ thus holding for $K > 0$. This is the case to which we restrict our analysis without loss of generality.) Therefore, we still have the two branches in configuration space which tend to the same limit $\beta = \beta_0$ in the low-energy regime $\bar{\phi} \to -\infty$.

What we called "pre-big-bang" branch ("post-big-bang" branch) is now the upper (lower) branch in configuration space, and the duality transformation transforms $\beta(\bar{\phi}) - \beta_0 \to \beta_0 - \beta(\bar{\phi})$. We note that the qualitative features of the trajectories remain unchanged if

we go back to the original configuration space variables β and ϕ, where ϕ is the original dilaton field, see Eq. (3). The two branches are then given by the equation (if we write $\bar{\phi} = \phi - \sqrt{3}(\beta - \beta_0)$)

$$e^\phi = |K|^{-1} e^{\pm(\beta-\beta_0)\sqrt{3}} \sinh(\pm\beta \mp \beta_0).$$

Coming back to the solution (7) of the effective equation (5) and its limit (8), one can see that

$$\lim_{\phi\to\infty} \Pi_{\bar{\phi}} J_{\pm ik}(z) = \mp k J_{\pm ik}, \tag{16}$$

where $\Pi_{\bar{\phi}} = -i\partial_{\bar{\phi}}$. This quantum relation, or more precisely, its analogy with the classical relation (13) was the key point in [10] to identify the two solutions $J_{\pm ik}$ with the "pre-big-bang" (J_{-ik}) and the "post-big-bang" (J_{+ik}) solutions, respectively.

As we have argued before, however, such a distinction has no intrinsic meaning. One can only talk about plane waves travelling *with respect to* the "intrinsic time" β, distinguishing small β and large β, but not *the* "pre-big-bang" and *the* "post-big-bang".

In order to gain further insight, we try to construct *wave packets* following the classical trajectories in configuration space given by (14). For the sake of this purpose it is convenient to study first a WKB approximation to (5). Since there are no classically forbidden regions, one has for all values of $\bar{\phi}$,

$$\psi_k(\bar{\phi}) \sim (-V_{eff})^{-\frac{1}{4}} \left[\exp\left(i\int \sqrt{-V_{eff}} d\bar{\phi} \right) + C \exp\left(-i\int \sqrt{-V_{eff}} d\bar{\phi} \right) \right], \tag{17}$$

where C is a constant, and "exp$(+)$" refers to "pre-big-bang", while "exp$(-)$" refers to "post-big-bang". The total WKB phase ($\psi_k \sim e^{iS_k}$) is then

$$S_k^{(\pm)}(\beta, \bar{\phi}) = -k\beta \pm s_k(\bar{\phi}), \tag{18}$$

where

$$s_k \equiv \int^{\bar{\phi}} \sqrt{k^2 + \lambda_s^2 \Lambda e^{-2\tilde{\phi}}} d\tilde{\phi}. \tag{19}$$

This integral can be solved exactly to give

$$s_k = \lambda_s \sqrt{\Lambda} \left\{ K \left[\mathrm{arsinh}(K e^{\bar{\phi}}) \right] - \sqrt{K^2 + e^{-2\bar{\phi}}} \right\}, \tag{20}$$

By the principle of constructive interference [14], the classical solutions are found through

$$\frac{\partial S_k^{(\pm)}}{\partial k} = -\beta \pm \frac{\partial s_k}{\partial k} = 0, \tag{21}$$

leading to Eq.(14) for the classical trajectories in configuration space. After rescaling $S_k^{(\pm)} \to S_k^{(\pm)}/\lambda_s \sqrt{\Lambda}$ we have for the total WKB phase (18)

$$S_k^{(\pm)} = -K\beta \pm K \mathrm{arsinh}(K e^{\bar{\phi}}) \mp \sqrt{K^2 + e^{-2\bar{\phi}}}. \tag{22}$$

In order to calculate wave packets for the two solutions (22) we take a Gaussian concentrated around $\bar{K} > 0$,

$$A_k = \pi^{-\frac{1}{4}} b^{-\frac{1}{2}} \exp\left[-\frac{1}{2b^2} \left(K - \bar{K} \right)^2 \right] \tag{23}$$

and consider the superposition

$$\Psi^{(\pm)}(\beta, \bar{\phi}) = \int_{-\infty}^{\infty} dK A_k \frac{e^{i S_k^{(\pm)}}}{(-V_{eff})^{\frac{1}{4}}}. \tag{24}$$

If the width b of the Gaussian is small enough, A_k is concentrated around $K \approx \bar{K}$, and therefore the integral (24) can be evaluated by expanding $S_k^{(\pm)}$ up to quadratic order in $K - \bar{K}$. Then,

$$i S_k^{(\pm)} = -i K\beta \pm i K \mathrm{arsinh}(\bar{K} e^{\bar{\phi}}) \mp i \sqrt{\bar{K}^2 + e^{-2\bar{\phi}}} \pm i \frac{(K - \bar{K})^2}{2\sqrt{\bar{K}^2 + e^{-2\bar{\phi}}}} + \ldots. \tag{25}$$

Inserting this into (24) and evaluating the resulting Gaussian integral, we have (choosing $\beta_0 = 0$ for simplicity)

$$\Psi^{(\pm)}(\beta, \bar{\phi}) = \sqrt{\frac{2}{\pi b}} \frac{1}{B} \left(\bar{K}^2 + e^{-2\bar{\phi}} \right)^{-\frac{1}{4}} \exp\left[-i\bar{K} \left(\beta \mp \mathrm{arsinh}(\bar{K} e^{\bar{\phi}}) \right) \right.$$
$$\left. \mp i\sqrt{\bar{K}^2 + e^{-2\bar{\phi}}} \right] \times \exp\left[\frac{1}{2B^2} \left(-\beta \pm \mathrm{arsinh}(\bar{K} e^{\bar{\phi}}) \right)^2 \right], \tag{26}$$

where

8

$$B^2 = \frac{1}{b^2} \mp \frac{i}{\sqrt{\bar{K}^2 + e^{-2\bar{\phi}}}}. \tag{27}$$

It is obvious that $|\Psi^{(\pm)}|^2$ is peaked around the classical trajectories (14) in configuration space. The absolute square of the width B is given by

$$|B|^2 = \frac{1}{b^2}\sqrt{1 + \frac{b^4}{\bar{K}^2 + e^{-2\bar{\phi}}}}, \tag{28}$$

so we have a very "mild spreading" of the packet.

We consider now packets from exact solutions. The strong coupling limit $\bar{\phi} \to \infty$ ($z \to 0$) was already performed in (8), while in the low energy limit $\bar{\phi} \to -\infty$ ($z \to \infty$) we have

$$J_{\pm ik}(z) = \sqrt{\frac{2}{\pi z}} \cos\left(z \mp \frac{\pi ik}{2} - \frac{\pi}{4}\right) + \ldots \propto \frac{1}{2} e^{\pm\left(\frac{\pi k}{2} - \frac{i\pi}{4}\right)} \left(e^{iz} + ie^{-iz \mp \pi k}\right). \tag{29}$$

The corresponding wave packets read

$$\Psi^{(\pm)}(\beta, \bar{\phi}) = \int_{-\infty}^{\infty} dK A_k e^{-ik\beta} J_{\mp ik}(z). \tag{30}$$

Following the discussion of [10] we note that after taking, for instance, the "pre-big-bang" solution $J_{-ik}(z)$ for $\bar{\phi} \to -\infty$ ($z \to \infty$), one has a superposition of $(+)$ and $(-)$ solutions (cf. Eq.(29)),

$$J_{-ik}(z) \propto e^{iz} + ie^{-iz} e^{\pi k} \equiv (-) + (+), \tag{31}$$

and therefore the relative probability between $(+)$ and $(-)$ is

$$\frac{|\Psi^{(-)}_{\bar{\phi}\to-\infty}|^2}{|\Psi^{(+)}_{\bar{\phi}\to-\infty}|^2} = e^{-2\pi k}. \tag{32}$$

However, in order to have a sensible wave packet, k should be concentrated around $k \gg 1$. This means that a "transition" into the $(-)$ component for $\bar{\phi} \to -\infty$ could only be interpreted as an extremely unlikely quantum effect in that region, but *not* as a transition into the other semiclassical component as represented by a wave packet. Roughly speaking, the $(-)$-component does not correspond to a "classical" trajectory if J_{-ik} is chosen as the exact solution.

To achieve interference between $(+)$ and $(-)$ wave packets, one must really *superpose* both packets,

$$\Psi = \alpha_1 \Psi^{(+)} + \alpha_2 \Psi^{(-)}, \tag{33}$$

i.e., choose

$$\Psi^{(\pm)}(\beta, \bar{\phi}) = \int_{-\infty}^{\infty} dK \, A_k e^{-ik\beta} \left[\alpha_1 J_{-ik}(z) + \alpha_2 J_{+ik}(z) \right] \tag{34}$$

with complex coefficients α_1 and α_2. This happens, for example, if boundary conditions are imposed in the low energy limit $\bar{\phi} \to -\infty$ instead of the strong coupling limit $\bar{\phi} \to \infty$, in contrast to [10]. (This is also the region where the effective theory can be trusted.) A superposition of J_{+ik} and J_{-ik} which corresponds to the $(+)$ solution for $\bar{\phi} \to -\infty$ (compare (31)) is the Hankel function

$$H_{ik}^{(2)}(z) \overset{z \to \infty}{\sim} \sqrt{\frac{2}{\pi z}} e^{-iz - \frac{\pi k}{2} + \frac{i\pi}{2}}. \tag{35}$$

Since

$$H_{ik}^{(2)}(z) = J_{ik}(z) - iN_{ik}(z) = (1 - \coth k\pi) \, J_{ik} + \frac{J_{-ik}}{\sinh k\pi}, \tag{36}$$

one finds that $H_{ik}^{(2)}(z)$ approaches in the strong coupling limit $\bar{\phi} \to \infty$ ($z \to 0$) the following asymptotic behaviour:

$$H_{ik}^{(2)}(z) \to \frac{1}{\sinh k\pi} \left[\frac{e^{-k\pi}}{\Gamma(1 + ik)} \left(\frac{z}{2} \right)^{ik} + \frac{1}{\Gamma(1 - ik)} \left(\frac{z}{2} \right)^{-ik} \right]. \tag{37}$$

The corresponding "transition factor" from $(+)$ to $(-)$ would then again be given by $e^{-2\pi k}$, but this time a second semiclassical component is indeed present. This is a generic feature if boundary conditions are imposed at $\bar{\phi} \to -\infty$: since the classical solutions overlap in this region, one finds in general a superposition of wave packets for $\bar{\phi} \to \infty$.

We want to include here a general discussion of boundary conditions in quantum string cosmology. If more than two degrees of freedom are present (which, of course, is the realistic case), the Wheeler-DeWitt equation is, at least for perturbations of Friedmann-type spaces

10

[19], hyperbolic with respect to β. One would thus expect to impose boundary conditions (Cauchy data) at $\beta = constant$ (or $a = constant$).

As long as one considers only minisuperspace degrees of freedom, the wave packets are just timeless wave tubes. A semiclassical time parameter, as well as the concept of a *direction* of time can only be defined if a huge number of further degrees of freedom ("higher multipoles") is present. A semiclassical time parameter emerges if the "background wave function" is in a WKB state [13]. Technically this is achieved by a Born-Oppenheimer type of expansion scheme, with the expansion parameter given by λ_s in the present case. It yields $\partial/\partial t \equiv \nabla S \cdot \nabla$ for background states $\psi_0 \approx e^{iS}$. A time direction then emerges from thermodynamical considerations if one starts from an uncorrelated state for $\beta \to -\infty$. Such an "initial condition" is facilitated by the fact that the potential term in the Wheeler-DeWitt equation vanishes in this limit (except for the dilaton part). Such an initial state can, for example, be of the form $\Psi = \psi_0(\beta, \phi)$, independent of other degrees of freedom (see [20] for the case of quantum general relativity). With increasing values of β, a correlated state would emerge, since the potential now depends explicitly on the higher multipoles. This in turn, leads to decoherence and increasing entropy for the background part (β, ϕ) [17,21].

For higher values of β one will then enter the semiclassical regime. One will then get, for example, a state of the form

$$\Psi \approx \alpha_1 e^{iS^{(+)}(\beta, \phi)} \chi^{(+)}(\beta, \phi, \{x_\lambda\}) + \alpha_2 e^{iS^{(-)}(\beta, \phi)} \chi^{(-)}(\beta, \phi, \{x_\lambda\}),$$

where $\{x_\lambda\}$ symbolically denotes all higher multipoles. It is then a quantitative question whether there will be also decoherence between these two components, in addition to the decoherence for each single component. It has been argued that there are regions in the (β, ϕ)-plane (concentrated towards negative dilaton values) where decoherence is ineffective [22]. It would nevertheless then be inappropriate to imagine this as a "transition" from one semiclassical component into the other, since the semiclassical approximation breaks down in such a region, so that no time parameter exists there. There thus exists no classical causal relationship between these branches. This makes it very hard to solve the "graceful

11

exit problem" in this framework, and one has to envisage alternatives such as in [7,8].

In the last part of our paper we want to discuss briefly some other possibilities for the dilaton potential $V(\beta, \bar{\phi})$ in the WDW equation (1) in order to gain insight into the problem of boundary conditions in other situations. First, we adopt (though hardly justified by string theory) the negative dilaton potential [9]

$$V(\beta, \bar{\phi}) = -V_0 e^{4\bar{\phi}} \qquad (V_0 > 0) \qquad (38)$$

in (1), which allows one to find the separable solution (4) with $\psi_k(\bar{\phi})$ obeying the effective equation (5). But now the effective potential is different from that of (6) and reads

$$V_{eff} = -k^2 + \lambda_s^2 V_0 e^{2\bar{\phi}}. \qquad (39)$$

The potential (39) leads to the existence of classically forbidden regions and "turning points". The key point of such a model is that the "pre-big-bang" and the "post-big-bang" branches are already connected at the classical level. It is, however, interesting that, in contrast to most situations in ordinary cosmology, it is not the scale factor a, but the shifted dilaton $\bar{\phi}$ which has a turning point. Another point is that the strong coupling limit $\bar{\phi} \to \infty$ is classically forbidden.

The corresponding classical self-dual solution [9] is given by

$$\bar{\phi} = -\frac{1}{2} \ln \left(\frac{V_0^{\frac{1}{2}}}{L^2} + L^2 V_0^{\frac{1}{2}} t^2 \right), \qquad (40)$$

$$\beta = \beta_0 + L^2 t + \sqrt{1 + L^4 t^2}, \qquad (41)$$

where

$$L = \frac{k}{\lambda_s V_0^{1/4}}. \qquad (42)$$

This solution is nonsingular at $t = 0$, and the evolution of the scale factor seems to describe a transition between the "pre-big-bang" accelerated branch and the "post-big-bang" decelerated branch of singular solutions like (9)-(12). After eliminating t from (42)-(43) we find

for the evolution equation in configuration space $(\beta, \bar{\phi})$ the equation[1]

$$\beta - \beta_0 = \pm \text{arcosh} \frac{e^{\bar{\phi}_0}}{e^{\bar{\phi}}}, \qquad (43)$$

where we have defined

$$e^{\bar{\phi}_0} \equiv \frac{L}{V_0^{1/4}}. \qquad (44)$$

The trajectory (43) cannot be divided into two branches which could be naturally interpreted as describing "pre-" or "post-big-bang" branches. In particular, the shifted dilaton has a "turning point" for $\bar{\phi} = \bar{\phi}_0$. This equation looks similar to the corresponding equation in the case of a massless scalar field in ordinary cosmology [14], except that the roles of field and scale factor are interchanged. For this reason we have here a "turning point" for the dilaton. As discussed above, β plays the role of an intrinsic time variable. A sensible boundary condition would then be to have a wave packet in the small β-region concentrated around a large negative value for the dilaton. This would then lead to a wave packet concentrated around the trajectory depicted in Fig 2. We emphasise again that this can only be interpreted as representing *one* cosmological solution. This solution can be labelled "expanding" only after a condition of low entropy is imposed for $\beta \to -\infty$ in the sense discussed above.

An interesting model with a turning point in β is obtained by taking into account a positive curvature term in the effective action [23]. We shall not, however, include a discussion of this model here, since the essential conceptual features remain unchanged.

------

[1]Note that there is also a self-dual solution which connects smoothly the "pre-big-bang" decelerated branch with the "post-big-bang" accelerated branch with the same Eq. (40) and the opposite sign before $L^2 t$ in (41), which leads to the same evolution equation in configuration space (43).

ACKNOWLEDGMENTS

REFERENCES

[1] E.S. Fradkin and A.A. Tseytlin *Nucl. Phys.* **B261** (1985) 1; C.G. Callan, D. Friedan, E.J. Martinec, and M.J. Perry, *Nucl. Phys.* **B262** (1985) 593.

[2] J. Polchinski, *Rev. Mod. Phys.* **68** (1996) 1245.

[3] G. Veneziano, *Phys. Lett.* **B265** (1991) 287; M. Gasperini, J. Maharana, and G. Veneziano, *Phys. Lett.* **B272** (1991) 277; *Mod. Phys. Lett.* **A8** (1993) 3701; *Astro. Phys.* **1** (1993) 317.

[4] R. Brustein and G. Veneziano, *Phys. Lett.* **B329**, 429 (1994).

[5] N. Kaloper, R. Madden, and K. A. Olive, *Nucl. Phys.* **B452** (1995) 677; *Phys. Lett.* **B371** (1996) 34.

[6] M. Mueller, *Nucl. Phys.* **B337** (1990) 37.

[7] M. Gasperini, M. Maggiore, and G. Veneziano, "Towards a Non-singular Pre-Big-Bang Cosmology", e-Print: hep-th/9611039.

[8] I. Antoniadis, J. Rizos and K. Tamvakis, *Nucl. Phys.* **B415** (1994) 497; R. Easther and K. Maeda, "One-loop Superstring Cosmology and the Non-Singular Universe", e-Print: hep-th/9605173, to appear in *Phys. Rev.* **D**.

[9] M. Gasperini, J. Maharana, and G. Veneziano, *Nucl. Phys.* **B472** (1996) 349.

[10] M. Gasperini and G. Veneziano, "Birth of the Universe as Quantum Scattering in String Cosmology", e-Print: hep-th/9602096, to appear in *Gen. Rel. Grav.* **28** (1996).

[11] H.D. Zeh, *The physical basis of the direction of time* (Springer, Berlin, 1992).

[12] J.B. Barbour, *Class. Quantum Grav.* **11** (1994) 2875.

[13] C. Kiefer, "The semiclassical approximation to quantum gravity", in: *Canonical gravity: From classical to quantum*, edited by J. Ehlers and H. Friedrich (Springer, Berlin, 1994),

pp. 170–212.

[14] C. Kiefer, *Phys. Rev.* **D38** (1988) 1761; *Nucl. Phys.* **B341** (1990) 273.

[15] M.C. Bento and O. Bertolami, *Class. Quantum Grav.* **12** (1995) 1919.

[16] M. Cavaglià and V. de Alfaro, "Time gauge fixing and Hilbert space in quantum string cosmology", e-Print: gr-qc/9605020.

[17] C. Kiefer and H.D. Zeh, *Phys. Rev.* **D51** (1995) 4145.

[18] H.D. Zeh, *Phys. Lett.* **A126** (1988) 311.

[19] D. Giulini, *Phys. Rev.* **D51** (1995) 5630.

[20] H.D. Conradi and H.D. Zeh, *Phys. Lett.* **A154** (1991) 321.

[21] D. Giulini, E. Joos, C. Kiefer, J. Kupsch, I.-O. Stamatescu, and H.D. Zeh, *Decoherence and the Appearance of a Classical World in Quantum Theory* (Springer, Berlin, 1996).

[22] A. Lukas and R. Poppe, "Decoherence in Pre-Big-Bang Cosmology", e-Print: hep-th/9603167; see also J.E. Lidsey, "Inflationary and Deflationary Branches in Extended Pre-Big-Bang Cosmology", e-Print: gr-qc/9605017.

[23] R. Easther, K. Maeda, and D. Wands, *Phys. Rev.* **D53** (1996) 4247.